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Teaching Mathematics Appreciation to Nonscience Majors

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I. WHAT DO I MEAN BY MATHEMATICS APPRECIATION?

As H. O. Pollak (35) very accurately observed, "the perception most people have of mathematics has been molded by their educational experience, and neither the experience, nor its recollection tends to be happy." Ironically, many people dislike and fear a mislabeled enemy, because they have only seen what David Fowler (18) calls schoolmath, a quite different subject with its own terminology, methods, and beliefs. Although it is true that most nonscience majors may have forgotten a good part of their schoolmath, the bad feelings about their experience remain.

On the other hand, in spite of the indisputable applicability of mathematics, it takes some planning to communicate the effectiveness of mathematics to a nonspecialized audience. "How can it be—Albert Einstein asked in 1920—that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?"

As a result of all these factors, the dealings of mathematics seem too often closed off as by a high wall. How do we breach this wall, how do we present mathematics in a way that a passerby may enjoy it? Better yet, how do we lure a reluctant spectator into becoming, to some extent, a performer? After teaching mathematics appreciation for several years to numerous nonscience majors, I am still struggling with these questions, although it seems that by the end of each semester I am able to come up with better answers.

I do not think that there is a universal recipe for what constitutes appreciation of mathematics. However, it is my experience that any successful approach should recognize the special characteristics and the wealth of nonscientific knowledge that the students have. To this purpose, I find it very effective to present mathematics in several combined ways: as a powerful tool in the students' own business, as a means to develop effective thinking and communication skills, as a phenomenon of cultural history, and as a collection of fundamental thoughts and ideas. I think that one of my tasks is to convincingly present the central role that mathematics has played in people's lives throughout history, and to emphasize the continuity and connectedness in the way mathematics develops. Another task is to convey what the physicist Eugene Wigner called "the unreasonable effectiveness of mathematics"

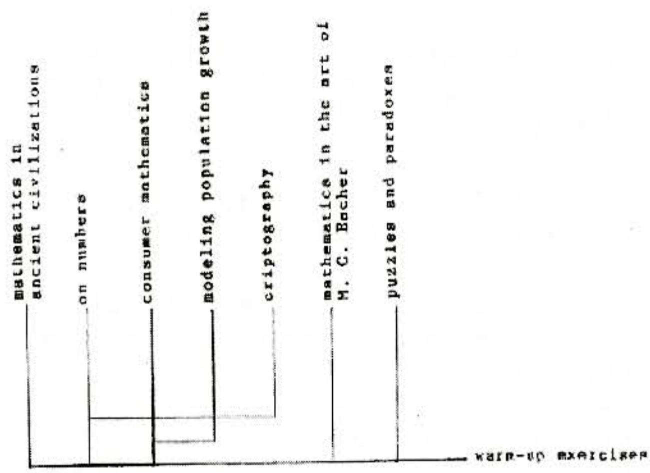
I do not think that a course on mathematics appreciation should insist on technical skills. Rather, it should make the best out of whatever skills the students have, typically a vague recollection of high school algebra. Still, I view this course as a mathematics course, where the students are expected to see and do mathematics. Many of the most beautiful and enduring mathematical ideas have a common sense quality that makes them fairly natural to grasp.

II. HOW DO I TEACH A BASIC COURSE ON MATHEMATICS APPRECIATION?

I do not require a textbook. Most of the books appropriate for this basic level, seem to be collections of methods and examples that fail to show how and why these methods arose. Instead, I provide the students with an extensive and eclectic list of materials gathered from scholarly books and journals, newspapers, magazines, etc. I am including a sample list at the end of this article.

What I have seen working best in my classes is to choose paths that go through a collection of topics that have some common thread whether in the subject itself or in the approach by which they can be presented. My purpose is to emphasize the continuity and connectedness in the way mathematics develops and progresses. For this purpose, it is crucial to emphasize credible beginnings, links to previous experiences or human needs. I keep reminding myself that, probably, this is the last chance to expose these students to the beauty and power of mathematics, that their feelings towards mathematics will undoubtedly influence their children's.

I start the semester with a set of simple problems that I call "Warm-up Exercises" and use them to preview what we will cover in detail later. These problems are the trunk from which the branches of the course will stem. Typically, I cover topics on mathematics in ancient civilizations, numbers, modeling population growth, consumer mathematics, cryptography, puzzles and paradoxes, and mathematics in the art of M. C. Escher. Here is a picture of such a course:



I show this picture to my students quite often along the semester, to emphasize the flow of mathematical events.

The class format is highly interactive, with students doing a lot of group work. For each topic, I prepare extensive handouts, including many historical references and hands-on activities. An overhead projector is frequently used to illustrate a topic. I require the students to have and use a scientific calculator. I prove and discuss on the blackboard some of the formulas to be used. Even very simple formulas, such as compound interest or annuities, provide powerful examples of mathematics at work. For most students, this is the first time they see how a formula is obtained. It also gives some credibility to the algebraic manipulations they may have seen before.

I limit my lecturing to an absolute minimum. My role in the classroom is more of a moderator and watcher of the students' discussions. I am fortunate to have a classroom with movable furniture that encourages and facilitates group work.

As part of the course requirements, the students have to prepare two group papers, and work individually on some in-class written essays and problems.

III. WHAT DID I LEARN FROM TEACHING MATHEMATICS APPRECIATION?

I have asked the students to answer brief questionnaires at the beginning and at the end of each semester. Their answers show a tremendous, positive change in their attitude towards mathematics. I quote here one of my favorites answers: "I expected drudgery of trying to complete copious amounts of stupid problems such as if two pipes fill a pool that is unplugged..." I am grateful to my students for many eye opening responses.

After teaching my first mathematics appreciation course in 1994, I wrote in my annual report: "This course opened up a completely new, challenging, and very rewarding world in my teaching career." I have taught since then several versions of the course and I still feel the same way.

Prior to teaching this course, I had frequently used group work and writing assignments in other courses. Teaching mathematics appreciation gave me the opportunity to improve my overall understanding of the historical development of mathematics. This newly acquired ability to seek and appreciate a bigger picture has made me a better teacher at the undergraduate and graduate level, as well as a better researcher, more able to look into the rich history of my own field of interest. I now make a special effort to look for supplemental reading materials for my students and myself that stress the historical development of a subject. And I am very pleased to find that more mathematical authors are trying to offer a more balanced picture between techniques and their historical development.

IV. SAMPLE LIST OF RESOURCE MATERIALS

1. Edwin A. Abbott, *Flatland, A romance of many dimensions*, Dover Publications Inc., New York, 1992.

2. Don Albers, *John Horton Conway: Talking a good game. The game of "Life"*, Math Horizons Spring 1994, MAA.

3. Marcia Ascher and Robert Ascher, *Ethnomathematics*, History of Science 24 (1986), 125-144.

4. Marcia Ascher and Robert Ascher, *Code of the quipu: A study in media, mathematics, and calture*, University of Michigan Press, Ann Harbor (1981).

5. Marcia Ascher, *Ethnomathematics, A multicultural view of mathematical ideas*, Brooks/Cole Publishing Co., California, 1991.

6. *Atlas of the World, Second Edition*, Oxford University Press, New York, (1994)

7. Ray Beauregard and E. R. Suryanarayan, *Pythagorean Triples: The Hyperbolic View*, The College Mathematical Journal 27 (1996), 170-181.

8. Petr Beckmann, *A history of pi*, The Golem Press, Colorado, (1971).

9. Carl Boyer, *Analysis: notes on the evolution of a subject and a name*, The Mathematics Teacher 47 (1954), 450-462.

10. Dario Castellanos, *The ubiquitous pi*, Mathematics Magazine 61 (1988), 67-98 and 148-163.

11. Levi L. Conant, *Counting*, The world of mathematics by J. R. Newman, Volume I, 432-441, Simon & Schuster (1956).

12. Linda Dalrymple Henderson, *The forth dimension and non-euclidean geometry in modern art*, Princeton University Press, Princeton, New Jersey, 1983.

13. P. J. Davis, *Applied mathematics as a social contract*, Mathematics Magazine 61 (1988), 139-147.

14. Keith Devlin, *Editorial*, Focus, MAA Newsletter, December 1993.

15. Keith Devlin, *Fermat's Last Theorem*, Math Horizons, Spring 1994, MAA.

16. Keith Devlin, Fernando Gouvea, and Andrew Granville, *Fermat's last theorem, a theorem at last*, Focus, MAA Newsletter, August 1993.

17. Bruno Ernst, *The magic mirror of M. C. Escher*, Barnes & Noble Books, New York, 1994

18. David Fowler, *What society means by mathematics*, Focus, MAA Newsletter, April 1994.

19. Joseph Gallian, *How Computers Can Read and Correct ID Numbers*, Math Horizons Winter 1993.

20. Martin Gardner, *Wheels, life and other mathematical amusements*, W. H. Freeman, New York (1983).

21. Kenneth N. Gilpin, *Market Takes Steepest Drop Since '91. The Dow Drops 171.24 After Jobs Surge*, The New York Times, March 9, 1996.

22. James Gleick, *Even Mathematicians Can Get Carried Away*, The New York Times, March 3, 1987.

23. J. V. Grabiner, *The centrality of mathematics in the history of western thought*, Mathematics Magazine 61 (1988), 220-230.

24. JoAnne Growney, *Mathematics in Literature and Poetry*, Humanistic Mathematics Network Journal 10 (1994), 25-30.

25. E. Hairer and G. Wanner, *Analysis by its history*, Springer-Verlag New York, Inc., 1996.

26. Gina Kolata, Andrew Wiles: *A Math Whiz Battles 350-Year-Old Problem*, Math Horizons Winter 1993, MAA. Reprinted from The New York Times, June 29, 1993.

27. Gina Kolata, *How a Gap in the Fermat proof was bridged at long last*, The New York Times, January 31, 1995.

28. Charles Krauthammer, *The Joy of Math*, or Fermat's Revenge, Time, April 18, 1988.

29. Steven Levy, *Scared Bitless*, Newsweek, June 10, 1996.

30. Robin Maconie, *The concept of music*, Oxford University Press Inc., New York, 1994.

31. R. M. May, *Simple mathematical models with very complicated dynamics*, Nature 261 (1976), 459-467.

32. Peter Monaghan, *The history of early Christianity undergoes sociological examination. A scholar offers new explanations for the growth of the religion*, The Chronicle of Higher Education, July 12, 1996.

33. Roger B. Nelsen, *Proofs without words: Exercises in visual thinking*, MAA (1993).

34. H.-O. Peitgen and P. H. Richter, *The beauty of fractals*, Springer, Berlin (1986).

35. H. O. Pollak, *Mathematics: the Science of Patterns*, book review, Notices of the AMS, November 1996, 1353-1355.

36. R. Preston, *Mountains of pi*, The New Yorker 68, March 2, 1992, 37-67.

37. Alan Sloan, *Dissing the Dow. The world's most famous stock indez turned 100 last week*, Newsweek, June 10, 1996.

38. David E. Smith and Jekuthiel Ginsburg, *From numbers to numerals and from numerals to computation*, The world of mathematics by J. R. Newman, Volume I, 442-464, Simon & Schuster (1956).

39. Dirk J. Struik, *The Sociology of Mathematics Revisited: A Personal Note*, Science & Society, Volume L, No. 3, Fall 1986, 280-299.

40. *The World Almanac and Book of Facts*, Funk & Wagnall Co., New Jersey.

41. A. Weil, *Number theory: An approach through history from Hummarapi to Legendre*, Birkhauser, Boston (1984).

42. H. Weyl, *Symmetry*, Princeton University Press, Princeton, New Jersey, 1952.

43. Alfred N. Whitehead, *Mathematics as an element in the history of thought*, The world of mathematics by J. R. Newman, Volume I, 402-417, Simon & Schuster 1956.

NUMBERS - 3

Numbers originate with who?
Did God or man define two?

Maybe Plato was right
Possibly a divine insight

Truths existing before time
In an ideal plane they reside

Numbers thus exist there
Not here or anywhere

However, its just possible
Numbers were definable

To appear mysterious timeless
That's speculation, I guess

Nevertheless
Numbers are what they are
Nothingless